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Research Article

GRAPH COLORING AND RELATED APPLICATIONS: THEORETICAL FOUNDATIONS, ALGORITHMS, AND PRACTICAL IMPLEMENTATIONS

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Abstract

Regarding scheduling, optimizing networks, and allocating resources, graph coloring—a fundamental notion in graph theory—is a game-changer. A thorough examination of graph coloring's theoretical underpinnings, methods, and real-world applications is provided in this work. This research provides a comprehensive overview of the area by examining ancient and new methodologies, such as greedy algorithms, backtracking methods, and heuristic-based strategies. In addition, the report assesses the efficacy of each method in various contexts and explores its respective limitations and intricacies. Graph coloring has several potential uses, including but not limited to assigning frequencies in wireless networks, allocating registers in compilers, analyzing social networks, and solving scheduling challenges. Hybrid algorithms and machine learning methods are two of the possible future possibilities suggested by the article, which also identifies present issues, including managing large-scale graphs and dynamic situations. Researchers and practitioners interested in improving efficiency and optimizing resource management using sophisticated graph coloring methods will find this detailed overview beneficial.

Keywords: Graph coloring, graph theory, algorithms, heuristic methods, network optimization, resource allocation, frequency assignment, register allocation, social network analysis, machine learning, hybrid algorithms, dynamic environments.

1. INTRODUCTION

Addressing optimization issues in the real world may include dealing with several forms of data, including classic unordered data, time series structured data, and graph-structured data. It is more difficult to successfully acquire knowledge in graph-structured data using typical machine learning models due to the extensive connection information included in the characteristics. Due to its exceptional capacity to extract relational information from data, graph neural networks (GNNs) have recently exploded in popularity in machine learning. Graph convolutional network (GCN) [1], GraphSAGE [2], graph attention network (GAT) [3], and many more are examples of the many GNN kinds that have been suggested; these networks are often classified according to the various aggregation techniques. Several fields have found success using GNN, including social networking [4], bioinformatics [5], and community discovery [6]. This is because GNN allows nodes to aggregate the knowledge of their neighbors. When you input the properties of nodes and edges and output the likelihood of each node belonging to distinct classes, you solve a bi- or multiclass classification issue. This is known as node classification. As part of the optimization procedure, each node creates embedding vectors by combining it's and neighboring nodes' information. Downstream machine learning tasks may employ embedding vectors to solve issues in either a non-autoregression or autoregression method [7]. These vectors represent node and edge characteristics in a higher dimensional space.

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The well-known Konigsberg seven-bridge issue was the starting point for graph theory [8-9]. The objective was to start anywhere along the Pregel River, cross each of the seven bridges once, and then return to the starting position. Euler [10] solved this problem using graphs in 1736. He used vertices to represent the land and edges to connect the two, with a bridge as a link. The issue was so graphically shown. For this particular issue, Euler discovered that no such closed walk existed. Consequently, the idea of an Eulerian circuit was introduced to graph theory, and the following statement was made: "A connected graph is Eulerian if the degree of all vertices is an even number and vice versa." So, the development of graph theory started with the publication of this first paper.

Subsequently, using the algebraic topology invariant of polyhedrons, L. Euler [10] created the formula for planer graphs. The equation $n + f - m = 2$ holds for every polyhedron P with n vertices, f faces, and m edges. The mathematical and physical communities owe great gratitude to L. Euler, who advanced graph theory, among other areas.

Graph theory gained prominence in 1850 with the famous four-color map. For about 127 years, we struggled to find a solution to this challenging challenge. Despite the best efforts of many academics and mathematicians, this issue remained unsolved.

2. LITERATURE REVIEW

Numerous areas of computer science use the graph coloring issue, such as frequency allocation, job scheduling, and resource optimization. Scientists have developed a plethora of algorithms and approaches to make the solution more efficient over the years. Part of our study includes researching alternative techniques and versions of greedy algorithms.

New heuristics for certain graph types, including interval graphs, are the subject of several articles, as [11] from 2013. The optimum greedy heuristic is presented in this study to guarantee that graphs are colored with the optimal number of colors in the optimal sequence. The authors conclude that this optimization allows for improved performance in certain network configurations, which, in turn, drastically simplifies the method in practical applications. Similarly, the greedy graph partitioning method is investigated in Ref. [12]. In big and complicated networks, where performance optimization is crucial, the authors find that greedy approaches are especially beneficial and propose using them in graphs with specified partitioning criteria.

Variants of greedy algorithms, such as b-greedy and z-greedy, are the subject of several articles, such as [13] from 2024. The authors conclude that these variations outperform standard greedy algorithms in highly connected networks. Complex graph topologies, where the ratio of vertex density to color plays a critical role, maybe more faithfully colored with the aid of these variations. Additionally, DSATUR and Welsh-Powell are two of several greedy algorithm versions compared in the 2016 study [14]. In cases when algorithm speed is critical, Welsh-Powell produces the best results, but the DSATUR method excels in handling dense graphs.

The performance of numerous approaches, such as First Fit, DSATUR, and Welsh-Powell, are compared and analyzed in various graphs in papers published in 2021, such as [15]. The authors find that while dealing with dense, complicated graphs, DSATUR performs best, and when dealing with smaller graphs, the Welsh-Powell method performs best. Contributing to our knowledge of the connection between graph structure and algorithm efficiency, this work stresses the significance of selecting a suitable algorithm according to graph features. Conversely, the publication [16] summarizes several algorithms using DIMACS benchmark graphs and finds that DSATUR uses the fewest colors and that Welsh-Powell is the quickest. Whether optimizing for time or amount of colors is more important, the authors suggest employing one of these techniques.

The 1992 study [17] provides further reading on the shortcomings of greedy algorithms, particularly when it comes to discovering intricate structures like cliques in massive networks. According to scientists, greedy algorithms cannot recognize such structures despite their speed. This suggests that more sophisticated approaches may be necessary in some instances.

3. GRAPH COLORING

The renowned four-color conjecture (4CC) started the practice of graph coloring in the middle of the nineteenth century. According to Francis Guthrie's discovery, every nation-state on the 1850 administrative map of England was painted in one of four colors. Every two neighboring countries were painted in a different hue. He discussed the matter with Frederick Guthrie, his brother. Afterward, Frederick brought up the issue with his professor, Augustus De-Morgan, who could not provide a solution. In 1852, Morgan probed William Hamilton about 4CC. afterward, in 1878; Arthur Cayley continued investigating the matter and posed a query to the London Mathematical Society.

To ensure that no two neighboring countries have the same hue, May [18] cites an essay by Harary [19] that states, "Any map on a plane or the surface of a sphere can be colored with on four colors." In 1879, Kemp [20] demonstrated the 4CC for the first time. However, Heawood [21] showed that the evidence was flawed and proved that the hypothesis was true for five colors. Proving 4CC has been a long-term goal of many mathematicians for almost a century. Proof of 4CC using the numerical technique for all maps with fewer than 40 nations was provided by Ore and Stemple [22] in 1969.

Meanwhile, the colorization of graph parts got underway. Lastly, in 1977, Apple and Haken [23] showed 4CC on a computer that had been run for 1200 hours. For the first time ever, a well-known mathematical issue was comprehensively resolved by computers. Numerous mathematicians have used other methods to confirm the 4CC proof. Using 633 irreducible reducible configurations, Robertson, Sanders, Seymour, and Thomas [24] demonstrated 4CC.

H. R. Bhapkar [25] showed this using a graph's PNR. Based on Gauss's basic algebraic theorem [27], which stipulates that any n-degree polynomial with a complex coefficient has exactly n zeros, Birkhofi [26] presented a Chromatic polynomial in 2012.

A large number of real-world issues may be illustrated using graphs. In graph theory, we find the answers to these difficulties. To name just a few examples, there are networks involved in transportation (e.g., roads, electrical systems with resistors, capacitors, and inductors), communication (e.g., Facebook, Twitter, and Instagram), signal-to-noise ratio (SNR) in signal transmission, organic molecule structure representation in chemistry, and many more. The collection of points is called vertices, and the lines linking some or all pairs of points, called edges, form graphs, which may be used to illustrate these structures.

4. GRAPH COLORING PROBLEM

A basic problem in combinatorial optimization, the Graph Coloring Problem (GCP) has many uses in fields as diverse as scheduling and network architecture. Vertex coloring in a graph using the fewest colors feasible while ensuring that no two neighboring vertices share a color is known as the Graph Coloring Problem (GCP). Computing the GCP is notoriously difficult for large-scale graphs since it is NP-hard. Because a precise polynomial-time algorithm does not yet exist. The good news is that the graph coloring issue has several heuristic techniques that can discover approximations of solutions. Scholars have investigated numerous heuristic and metaheuristic methods to tackle this intricate optimization issue. The Genetic Algorithm (GA) is a promising method; it takes its cues from evolution and natural selection. Genetic algorithms are well-suited to addressing combinatorial issues like GCP because of their capacity to manage large search areas and provide optimum answers.

Graph coloring is useful in various contexts, such as:

- Scheduling: Using graph coloring, you may avoid scheduling jobs that are incompatible with one another. For example, graph coloring may be used to arrange school courses to ensure that no two students are enrolled in concurrent sessions.
- Register allocation: Compilers may use graph coloring to avoid assigning concurrently utilized

variables to the same register.

• Channel assignment: Wireless network channels may be assigned using graph coloring to prevent simultaneous usage of two adjacent channels.

To address the graph coloring problem (GCP), the authors of this study—Dey, Arindam et al.—introduce a genetic algorithm (GA). The authors suggest An innovative GA designed to tackle the GCP effectively [1]. The suggested GA shows competitive performance regarding solution quality and execution time via extensive experimental evaluations. By presenting a GA-based approach for the GCP and offering insightful analysis of the algorithm's performance on large-scale graph examples, the research makes a substantial contribution to the area. S. Balakrishnan et al. [28] suggest a MOGA or multi-objective genetic algorithm to tackle the issue of graph coloring. Minimizing the number of colors and maximizing other key graph features are two of the many competing aims that the authors acknowledge are involved in the GCP. The MOGA optimizes all of these goals at once, giving you a bunch of solutions that show you how to trade off some of them. Experiments on several graphs show that the MOGA method yields various good answers. This article explains how GCP may benefit from multi-objective optimization and highlights the many advantages of using GA-based solutions.

For the issue of graph coloring, Ardelean et al. presented a new GA solution using a Reduced Quantum Genetic Algorithm [29]. To improve search space exploration, they provide a novel method that keeps the population size constant throughout. The fixed-size pool approach enhances the convergence speed and solution quality compared to conventional GAs. Computing time and solution quality are two experimental metrics used to measure the graph's performance relative to benchmark graphs. In order to address the graph coloring issue (GCP), idi Mohamed Douiri et al. [30] suggested a hybrid strategy. The authors proposed a hybrid method to depict graph colors that allows for successful exploration of the solution space. We used advanced local guided search to assess the method's efficiency in producing high-quality results while keeping computational overhead to a minimum. By presenting a new encoding approach and demonstrating the possible uses of recorded GAs for the GCP, the article makes substantial contributions to the field. Marappan et al.'s [31] enhanced genetic algorithm for the GCP incorporates a new crossover operator that relies on complementary solutions. The suggested operator encourages search space expansion by allowing solutions to share supplementary genetic information. Extensive testing on several graph situations shows that the unique crossover operator considerably boosts solution diversity and convergence speed. The algorithm's advantages in terms of execution time and solution quality are shown by comparing its performance with those of classic GAs. In order to tackle the GCP, X. Li et al. [32] suggest a hybrid method that merges a genetic algorithm with a variable neighborhood search. The hybrid approach employs VNS as a local search technique to enhance the GA-generated solutions. The method efficiently searches the solution space and refines solutions locally by embedding the VNS inside the GA's evolutionary framework. Compared to standalone GAs and VNS approaches, experimental graph evaluations show that the hybrid strategy performs better. Highlighting the potential of hybrid techniques for the GCP, generating competitive performance in graph coloring challenges, and displaying the synergy between VNS and GA are all noteworthy contributions of this study.

For the rotating seru manufacturing challenge, Feng Liu et al. [33] present a dynamic multi-objective genetic algorithm. In order to strike a balance between exploring and exploiting the solution space, the technique employs a hybrid crossover operator that combines several crossover strategies. In order to increase convergence and solution variety, the algorithm continuously adjusts its parameters during development. This work emphasizes the value of dynamically altering GA settings and adds to the literature on multi-objective optimization methods. Feng et al. analyzed several genetic algorithm techniques based on the chromatic number they produced. Using the correct GA technique allows quicker convergence with a minimal population size, as the authors have emphasized—the number 34. Results from graph-based experimental assessments show that the GA operator significantly improves the algorithm's performance. The genetic algorithm outperforms traditional GAs by using the graph's intrinsic structure. This study explores the problem-specific operators for the GCP and provides useful insights into how genetic algorithms might be tailored to solve optimization challenges. In their groundbreaking work on optimization, Mohamed et al. provide a genetic algorithm-based solution. More effective optimization issue solving is the goal of the authors, who provide a novel Gaining Sharing Knowledge method [35]. The technique promotes successful search space exploration by using unique genetic operators adapted to the particular encoding. Optimization experiments show that the suggested method outperforms the state-of-the-art regarding computing time and solution quality. An innovative genetic algorithm, GSK, is introduced in this work for optimization, making a significant contribution to the area. Graph coloring is an NP-hard issue, and R. Marappan et al. addressed the robust approach to solving it. This research offered a strategy for introducing the conflict gene mutation and the single-parent model [36]. In addition, it found major improvements by comparing the time required for the conversion to the previous approaches. Et al. detailed the outcomes after applying evolutionary algorithms to the graph coloring issue. This work used factors such as size, topology, and edge density to compare the suggested technique with classic GAs [37]. The findings show that the EA gives superior statistics compared to standard algorithms. Using a parallel technique incorporating HPGA, Eiben et al. [38] provide a novel way to solve the GCP. The suggested approach, which is based on VOA, was addressed in this study. They compared the new technique to the conventional methods using the DIMACS website and found that it was superior. Once again, to address the issue of the single conflict gene, R. Marappan et al. (2016) suggested a novel method to resolve it [39]. Compared to the previous study and contemporary GA, it produces far better outcomes. In order to address the GCP, Kong Y. et al. suggested the RGCP technique, which applies the method to provide a practical solution to the issue. They also suggested an application that might be developed for schedule management [40].

5. VERTEX-COLORINGS AND EDGE-COLORINGS

An example of a vertex-coloring function would be given a set C of colors, anything from numbers to letters to names. The function would then assign a value from C to each vertex in the graph. In a perfect world, neighboring vertices would never have the same color assigned to them. Similarly, an edge coloring function uses a collection of colors C to give each edge in a graph a value.

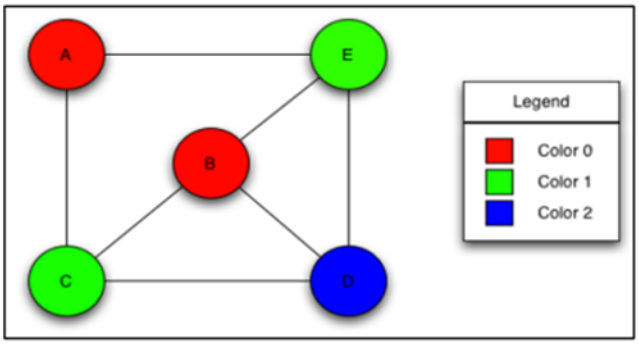

Fig 1: Vertex-Coloring Graph

At its most basic, graph coloring is a method for coloring vertices in a graph in a manner that prevents neighboring vertices from sharing the same color. Similarly, edge coloring makes sure that no two incident edges have the same color by assigning a different color to each edge.

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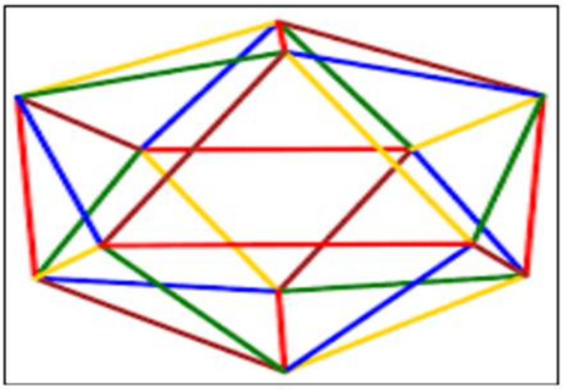

Fig 2: Edge-Coloring Graph

Region Coloring: "region coloring" involves giving distinct colors to different parts of a planar graph so that no two nearby areas share a hue. In geographic terms, two areas are considered neighboring if they share an edge.

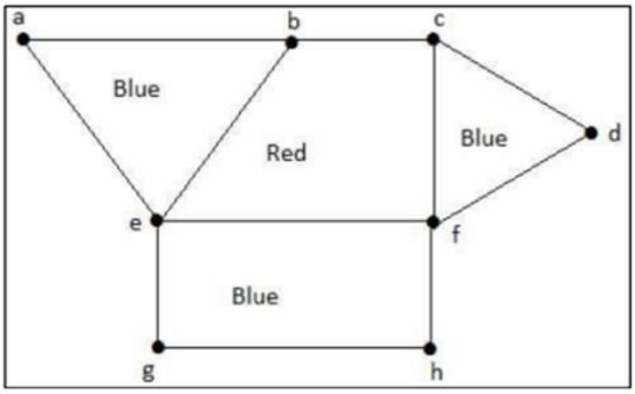

Fig 3: Region coloring Graph

6. K-COLORABLE

A graph G that is k-colored is labeled at the vertex level by the function f: V (G) → S, where S is a set with a cardinality of k. S is often represented as {green, blue, red, yellow, etc.} or, more abstractly, as the non-negative integers S = {1, 2, k}, or as a set of k distinct colors. Colors are the names of the labels. If the colors of neighboring vertices are distinct, then a k-coloring is correct. When the k-coloring is correct, we say that the graph is k-colorable. For each positive integer k, the chromatic number χ (G) is the smallest value that makes G colorable up to k. Being k-partite is the only condition for a graph to be called k-colorable. Put, k-colorable, and kpartite are the same terms. Identifying the k distinct sets of parts of a k-colorable graph and, conversely, finding the k colors of a k-partite graph. Finding a graph's chromatic number or determining if it is colorable for a given value of k is often not simple.

7. GRAPH COLORING ALGORITHMS

An algorithm is a series of instructions or rules that indicate how to execute a job step by step to produce the desired results. The term algorithm denotes "a set of rules or process to be followed in calculation or other solving problems."

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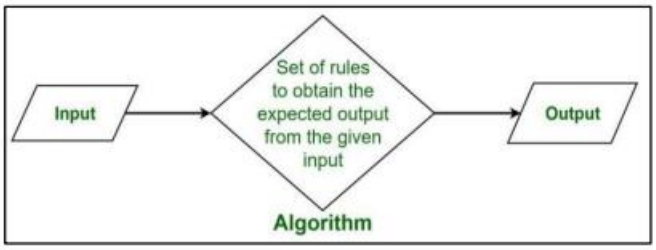

Fig 4: Graph coloring algorithms

8. GREEDY COLORING

Assigning the lowest available shade not utilized by v1's neighbors among v1....-1, the greedy method looks at the vertices in a certain sequence $a1 \ldots\ldots an$ and adds a new color if needed. The chosen purchase method determines the coloring quality. A greedy coloring with the optimal amount of x (G) colors may be achieved by following a certain sequence. Conversely, greedy colorings may be arbitrarily nasty; for instance, a two-colored crown network with n vertices can have an ordering that results in greedy coloring with n/2 colors. Optimal colorings for chordal graphs and certain instances of such graphs may be found in polynomial time using the greedy coloring approach, which involves choosing the vertex ordering as the inverse of a perfect deletion ordering. It is NP-hard to find an ideal ordering of fully orderable graphs extending this characteristic. Similar to greedy coloring, several additional graph coloring heuristics (also called sequential coloring algorithms) rely on a static or dynamic ordering of the vertices. A graph's Grundy number is the highest number of colors the greedy method can produce using vertex ordering to maximize this number.

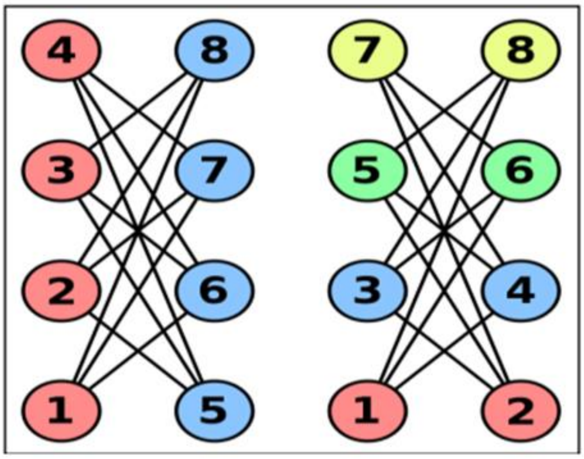

Fig 5: Bipartite Graph Color

It is possible to compute in linear time using breadth-first or depth-first search whether we can determine if a graph may be colored with two colors; this is the same as checking if the graph is bipartite. In a broader sense, the chromatic number and a matching coloring of ideal graphs may be calculated using semidefinite programming in polynomial time. It is possible to evaluate several types of graphs in polynomial time, including forests, chordal graphs, cycles, wheels, and ladders, for which closed formulae for chromatic polynomials exist.

9. DECENTRALIZED ALGORITHMS

In contrast to distributed algorithms, which permit message transmission at the local level, decentralized algorithms prohibit it altogether. Some efficient decentralized algorithms can color a graph, provided the appropriate coloring is available. These functions work because a vertex can detect whether its neighbors also use the same color or if there is a local dispute. Assuming a station can identify whether other transmitters are utilizing the same channel (e.g., by measuring the SINR) is an acceptable assumption in many applications; for example, when allocating wireless channels, it is often appropriate to assume this. With this amount of sensing data, learning automata-based algorithms may, with a one-in-ten chance, determine the correct graph coloring.

10. RECURSIVE LARGEST FIRST (RLF)

The RLF method is among the most well-known greedy strategies regarding the vertex coloring issue. It builds color classes successively using greedy selections. Notably, the first vertex set in a color class C is selected based on its largest number of uncolored neighbors. Subsequent vertices set in C are selected to ensure they have an equal number of uncolored neighbors that cannot be in C. Selections like these, driven by greed, may significantly impact the algorithm's performance—polynomial duration.

11. DECATUR ALGORITHM

The Dsatur algorithm, like the Greedy algorithm, gives more weight to vertices that are perceived as "most constrained"—that is, vertices with the fewest available color options—after selection. This is done by assigning the selected vertex to the lowest color label that has not been assigned to any of its neighbors. Since less constrained vertices may be colored later, these "more constrained" vertices are handled first. Unlike the greedy method, the Dsatur algorithm may be expected to employ a more predictable amount of colors due to its execution-generated vertex ordering. Compared to the greedy method, its solutions also often use fewer colors. If a network has more than one component, the algorithm will color all the vertices before moving on to the next component. Several graph topologies, such as bipartite, cycle, and wheel graphs, are also accurately covered by Decatur.

12. GENETIC ALGORITHM

In the context of Darwinian and evolutionary biology, a genetic algorithm is a computational model attempting to replicate natural selection's effects. Mathematical modeling is required prior to issuing a solution. A chromosome represents a potential solution. Multiple components, each known as a gene, make up each potential solution. To begin with, the algorithm will randomly create a population of potential solutions, also known as the first generation of chromosomes. Then, each chromosome's fitness and the likelihood of its selection in the following development are determined by the fitness function, which is used to develop the fitness of these chromosomes. The following event is the crossover. Crossover occurs when two chromosomes from different parents are found, cut at the same spot, and then joined to create a new chromosome. The genes of the father and the mother are mixed in the new chromosome. Through the process of crossover, n-m new chromosomal segments will be generated. The n-m newly-arisen chromosomal segments will undergo variation upon crossover. The goal of variation is to pick optimal genes by rearranging their combination orders to bypass the algorithm's current search limit and yield the optimum answer. Finally, the fittest chromosomes from one generation are passed on to the next in an unaltered form when m pieces of new chromosomes are produced by copying. Each development requires the formation of n chromosomes, each evolution requires the generation of n-m chromosomes via crossover, and the remaining m chromosomes are formed by copying the best quality m chromosomes from the preceding generation.

13. APPLICATIONS OF GRAPH COLORING

Since graph theory addresses practical issues and their solutions, it finds widespread usage (for more information, see Narsingh Deo [41], Roberts [42], and Berge [43]). Without a doubt, mathematical applications of graph theory include the resolution of signal ow problems and other issues about linear systems. In statistics and probability theory, the Markov process is crucial for analyzing data from various sources, including computer programs, genetics, inventories, control theory, and control theory. Consequently, Markov processes are solved using graph theory. Chemical structures are represented and matched using graph theory in chemistry. Novel chemical composites may be identified and described using graph enumeration methods.

The two most important parts of computer engineering are program design and analysis. Program execution time and storage needs estimate error detection, program segmentation and flow, and program stochastic model creation is all areas where graph theory is used in computer programming. Other applications include optimizing programs, creating automated ow charts, representing data as a graph, and finding the validity and equivalence of programs by converting their digraph to a canonical form.

Graph coloring has recently attracted much attention again because of its practical uses (for example, check [42] for the most current research). Graph coloring can solve many practical scientific problems, such as computer networks, artificial intelligence, machine learning, electrical circuits, and communication networks. These fields are all part of computer engineering, electrical engineering, machine learning, and communication and electronics. Frequency allocation is a notable example of one of these uses. Each radio transmitter in a network uses a predetermined set of frequencies. Interference may occur when two nearby emitters use the same frequency. According to the most basic paradigm, these transmitter pairs should have been given separate frequency bands. The goal is to lower the total frequency employed. The graph coloring resolves this issue. When an edge connects two potentially interfering sets of emitters (vertices), we say that they are emitters in this context. So, the vertices' colors correspond to their frequencies, and adjacent vertices cannot have the same color. Larger systems often aim to minimize variance between the lowest and highest frequencies since the needed frequency separation increases with closer transmitter pairs or more frequency bands allocated to a single transmitter.

According to Hale [44], it was generally believed that frequencies should be discrete, evenly spaced spots on the spectrum. Therefore, colors are often thought of as numbers.

Later, from Hale's article, a T-coloring model and a channel assignment model for frequency assignment are also derived. Afterwards, utilizing the idea of list coloring, Tesman [45] created a list T-coloring model for assigning frequencies, which is constrained by the available frequencies for the transmitter.

Bhapkar [46] detailed a security key generator that uses a perfect weighted planar network in 2018. This publication also details the technique for generating public and secret keys.

Bhapkar et al. [47] provided a detailed explanation of viral graphs and their application in 1920.

Virus graphs come in four varieties, numbered I through IV; types I and II do not cause mortality, while kinds III and IV are very dangerous to humans. This research also covers the significance of graph modeling in the context of pandemic circumstances and the velocity of its spread. Isolating individuals using the cut-set approach helps restrict the spread of the COVID-19 virus.

Collaborating on brain MRI separation in 2022, Ghorpade and Bhapkar [48-49] used the cut-set idea to pinpoint the precise contaminated region, aiding medical therapy. They spoke about using cut and watershed models for brain MRI segmentation.

14. CONCLUSION

A fundamental issue in graph theory, graph coloring has broad practical and scientific implications. In this study, we have covered the fundamental theoretical concepts, compared the effectiveness of several algorithms, and looked at real-world applications. According to the research, heuristic and metaheuristic techniques perform better in complicated situations, while greedy algorithms and backtracking provide baseline answers. Furthermore, the report stresses the need to deal with new problems, such as managing massive graphs and adjusting to ever-changing settings. Future research initiatives should focus on hybrid algorithms that use machine learning methods for predictive and adaptive graph coloring. These algorithms should incorporate the capabilities of multiple methodologies. Researchers may improve decision-making and resource management in many contexts by progressing in these areas. In the end, this extensive study hopes to direct future studies and contribute to the continuous development of graph coloring methods.

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